Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5712505207

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

[Turn over

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1 **(a)** Find the first three terms in the expansion, in ascending powers of x, of $(1 + x)^5$. [1]

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(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^6$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(1 + x)^5(1 - 2x)^6$. [2]

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2 By using a suitable substitution, solve the equation

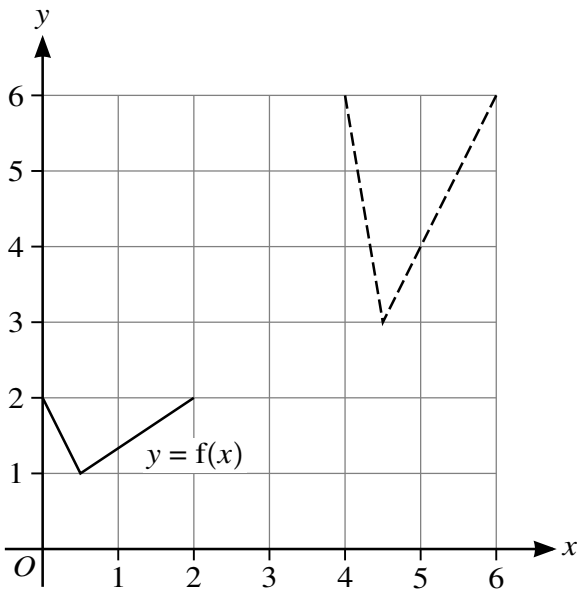
$$(2x - 3)^2 - \frac{4}{(2x - 3)^2} - 3 = 0.$$ [4]

3 Solve the equation $\frac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

4 A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

5



In the diagram, the graph of $y = \mathrm{f}(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

(a) Describe fully the two single transformations of $y = \mathrm{f}(x)$ that have been combined to give the resulting transformation. [4]

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(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

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6 A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\,(1, -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

(b) Find the equation of the curve. [4]

7 Functions f and g are defined as follows:

$$\text{f} : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$\text{g} : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $\text{f}(x)$ in the form $(x + a)^2 + b$ and state the range of f. [3]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

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(c) Solve the equation $\mathrm{gf}(x) = 13$. [3]

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8 The points $A(7, 1)$, $B(7, 9)$ and $C(1, 9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

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(b) Find an equation of the tangent to the circle at B. [2]

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9 The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\dfrac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

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(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

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(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

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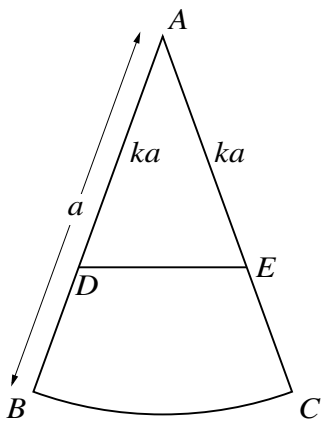

The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

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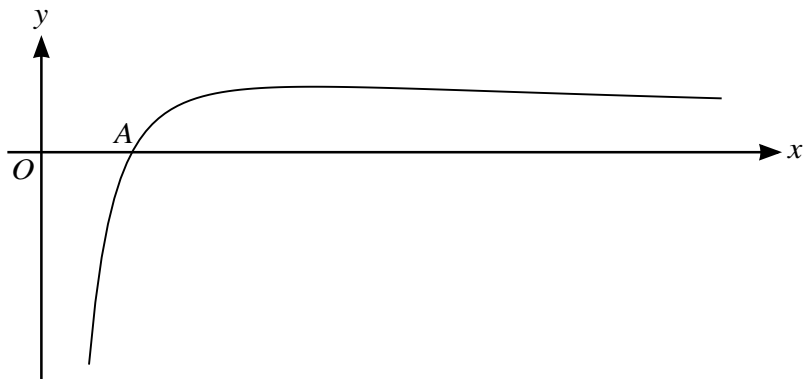
(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\dfrac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

11



The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

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(b) Find the equation of the tangent to the curve at A. [4]

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(c) Find the x-coordinate of the maximum point of the curve. [2]

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(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

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Additional Page

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